Test 2 - MTH-1400 Online

Dr. Adam Graham-Squire, Summer 2019

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Don't panic.
2. A computer is required to do the Honorlock online proctoring. If you have not already set that up, you should do so now. You should not use the computer for anything other than the proctoring.
3. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points. Clearly indicate your answer by putting a box around it.
4. Cell phones, notes and textbooks are not allowed on this test. Calculators are not allowed on the first 5 questions of the test. Calculators are allowed on the last 5 questions, however you should still show all of your work. You will initially receive the entire test, and you will NOT be allowed a calculator. Once you have finished everything you can without a calculator, you should turn in the first part of the test (the first 5 questions) to the proctor. The proctor can then give you your calculator and you can finish the remaining questions. You are not allowed to go back to the No Calculator portion once you have been given your calculator.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Note that for some questions your calculator will need to be in degrees mode, and others it will need to be in radians.
7. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
8. If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.
9. Make sure you sign the pledge.
10. Number of questions = 10. Total Points = 55.

1. (9 points) Without using a calculator, calculate the following. If the expression does not exist or is undefined, say so and explain why. Make sure to show your work, if there is any work to be shown! If it would help you, you can fill out the unit circle on the next page.

   (a) $\cos(60^\circ)$

   (b) $\sin\left(\frac{7\pi}{4}\right)$

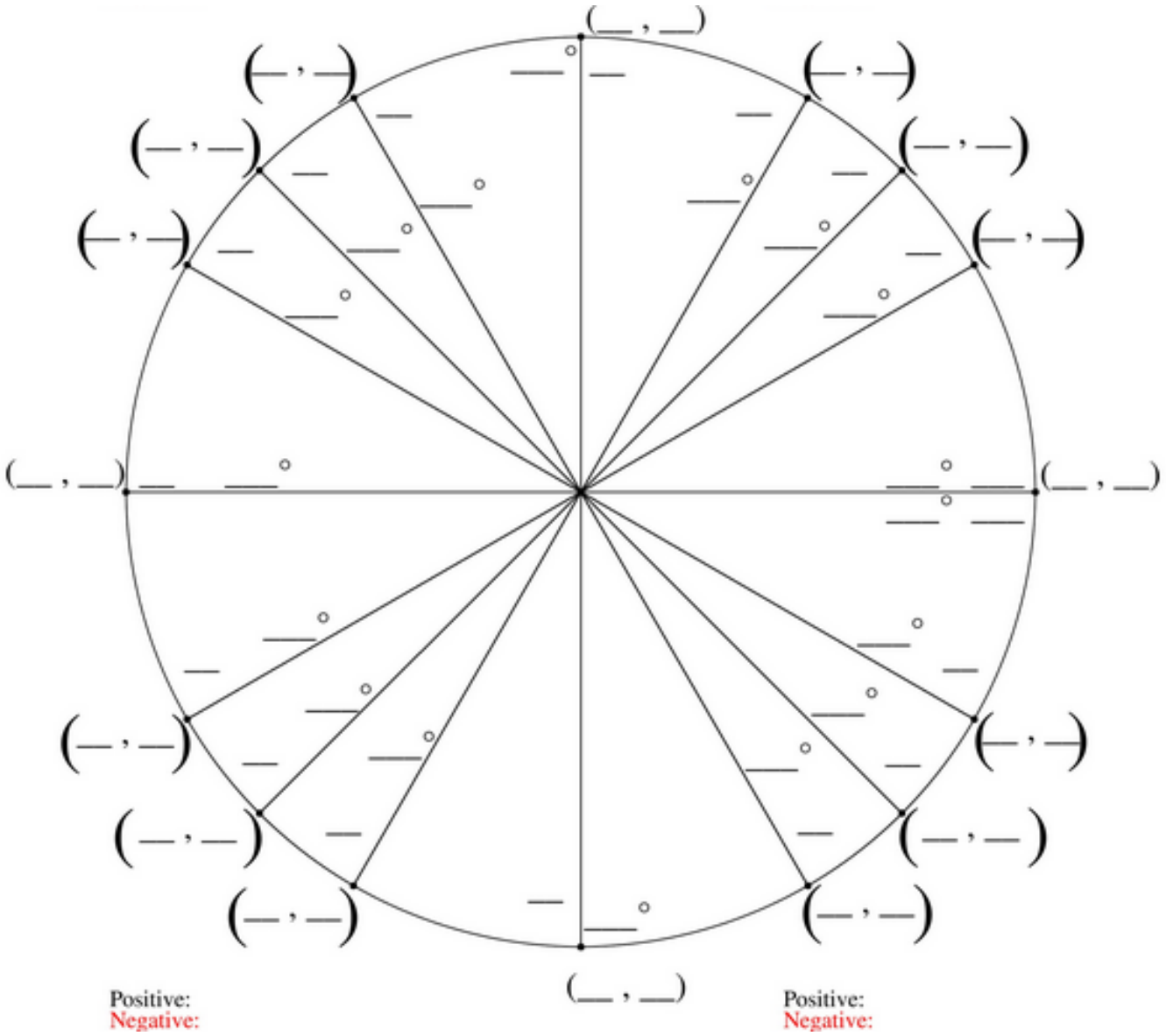
   (c) $\csc(-540^\circ)$

   (d) $\cot\left(\frac{41\pi}{6}\right)$

   (e) $\sin^{-1}\left(\frac{2}{\sqrt{2}}\right)$

   (f) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Unit Circle for you to fill in, if you would like to:



2. (3 points) For the equation $\cos\theta = \frac{-1}{2}$, find 4 different solutions in radians or degrees: one negative solution, two solutions between 0 and 2π, and one solution greater than 2π.

3. (3 points) Solve the equation

$$\log_3(x+8) + \log_3(x) = 2$$

either algebraically or graphically.

Whichever method you choose, you should explain/show your work.

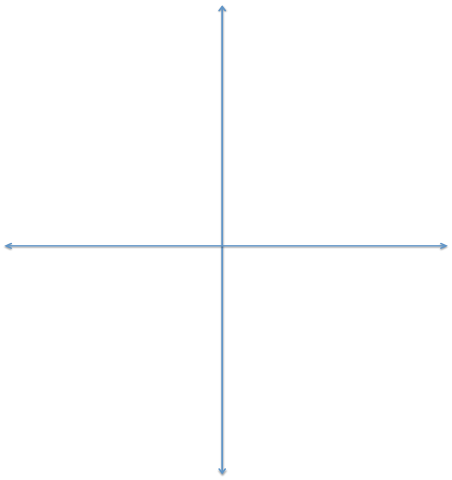
4. (5 points) (a) Starting with the identity $1 + \cot^2\theta = \csc^2\theta$, multiply both sides of the equation by $\sin^2\theta$ to find the trigonometric identity relating the values of $\sin\theta$ and $\cos\theta$. Show your work.

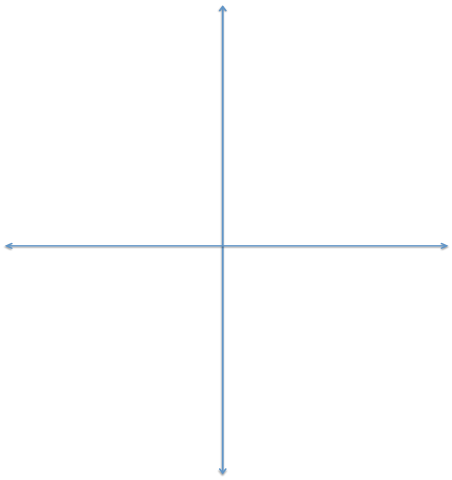
(b) If $\sin\theta = \frac{-1}{4}$ and θ is in the fourth quadrant, use trigonometric identities to find the exact values (no decimal approximations) of (i) $\cos\theta$, (ii) $\sec\theta$, and (iii) $\cot\theta$.

5. (5 points) The graph of the arctangent function ($y = \arctan x$ or $y = \tan^{-1} x$) is unusual because it has two different horizontal asymptotes.

   (a) *State* what the two different horizontal asymptotes are (one for infinity and the other for negative infinity) and

   (b) *Explain* why those are the horizontal asymptotes. You will likely need to sketch the graph of $y = \tan x$, and discuss inverse functions, in order to give a good explanation.





With Calculator Portion Name:_____

•Note that once you get your calculator and turn in the No Calculator portion, you CANNOT return to that part of the test!

•If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.

•Your calculator should be in radians for problem 7, and *degrees* mode for all remaining questions.

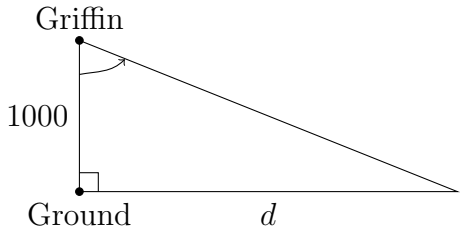
6. (5 points) (a) Suppose the element Mongoosium has a half-life m. Starting with the general form of the exponential growth/decay model $A = Pe^{rt}$, prove that $r = \dfrac{\ln(1/2)}{m}$ (or, equivalently, $r = -\dfrac{\ln 2}{m}$). (Hint: it may help to consider a specific example, say you have 100 grams of Mongoosium, and then do the necessary calculations from there.)

(b) Suppose the half-life of Mongoosium is $m = 1732$ years. A staff made of Mongoosium is unearthed at a geological site. The staff has only 6% of the Mongoosium left of the original amount. How long ago was the staff buried? Round to the nearest year.

7. (5 points) Griffin is standing on the roof of the Smith tower (1000 feet high), and they attach a laser pointer to their fidget spinner, and spin it slowly counterclockwise, so that the laser pointer makes a full rotation every 4 minutes. At time t, the distance d along the ground (from the base of the tower to the laser dot on the ground, as shown in the diagram below) is given by

$$d(t) = 1000 \tan\left(\frac{\pi t}{2}\right)$$

where d is measured in feet and t in minutes. Note: your calculator must be in *radians* mode for this function. Also, at $t = 0$, the light is pointed straight down, so $d = 0$.



(a) Find the distance d at the given times: (i) $t = 0.1$ and (ii) $t = 0.7$. Round answers to nearest whole number.

(b) How long does it take for d to reach 3000 feet? Explain/show your work, and round to nearest 0.001

(c) What happens to d as t gets closer to 1 minute? Explain why this makes sense *both* by looking at the function *and* by what would happen in the diagram (real life).

8. (10 points) Pat is sitting in a tree, looking straight across to a neighbor's house, which is x feet away (in horizontal distance) from Pat. The top of the neighbor's house is higher than Pat, and Pat measures the angle of elevation to the top of the neighbor's house to be $17°$. The ground is below Pat, and Pat measures the angle of depression to the bottom of the neighbor's house to be $39°$.

   (a) Draw a diagram of the situation.

   (b) If the distance x between the houses is 20 feet, calculate the height h of the neighbor's house. Round to nearest 0.01 feet.

Continuation of problem 8:

Pat is looking out a window of their house, straight across to Pat's neighbor's house, which is x feet away (in horizontal distance) from Pat's house and h feet tall. The top of the neighbor's house is higher than Pat, and Pat measures the angle of elevation to the top of the neighbor's house to be 17°. The ground is below Pat, and Pat measures the angle of depression to the bottom of the neighbor's house to be 39°.

Draw a diagram of the situation (should look same as in previous problem, part (a), but you *cannot* assume that x is still equal to 20).

(c) Now suppose you do not know the distance x between the houses, but you *do* know that the house is 32 feet tall. Can she calculate the distance x? If not, explain why not. If so, calculate it (round to nearest 0.01) and explain how you got your answer.

9. (5 points) A 47 foot long ladder leans to touch the top of a building that is 36 feet tall. What is the angle of elevation (in degrees) of the ladder? In other words, what is the angle between the ladder and the ground? Round your answer to the nearest 0.01.

10. (5 points) For the given triangle ABC, do the following:

(a) determine if there is <u>one</u>, <u>multiple</u>, or <u>no</u> ways to solve the triangle.

(b) If there is one way to solve the triangle, find the length of the third side. If there are *multiple* ways to solve the triangle, find the length of the third side in the triangle that has an *obtuse* angle.

Round answers to the nearest 0.01, and explain your reasoning/show your work.

$\angle B = 38.9°$, $c = 109.7$, and $a = 73.2$

Extra Credit (up to 2 points): You can either put 2 or 0.5 for how many extra credit points you want. If you put 0.5, you are guaranteed to get that half point. If you put 2, and you are the only student in the class who puts a 2, then you get 2 points. If more than one person puts a 2, though, everyone who put a 2 gets zero extra credit points. Note: there are 6 people in the class.